# The K value enumeration of hydrocarbons alkanes, alkenes and alkynes 

E.M.R. KIREMIRE<br>Department of Chemistry, University of Namibia, Private Bag 13301, Windhoek (Namibia).

(Received: September 20, 2008; Accepted: November 02, 2008)


#### Abstract

Isomerism is a powerful concept in chemistry. The use of the K value greatly simplifies the sketching of structural isomers in hydrocarbons. The calculation of the K value has been demonstrated. The number of isomers and K values of selected hydrocarbons have been tabulated. The K value complements the octet rule. It is hoped that the use of K value will greatly enhance the teaching of structural isomerism and molecular geometry in organic compounds at undergraduate level.


Key words: k value, hydrocarbons, alkanes, alkenes and alkynes.

## INTRODUCTION

Isomerism in alkanes has attracted the attention of many researchers for about 150 years ${ }^{1}$. It is exhibited in all the alkanes starting with butane, $\mathrm{C}_{4} \mathrm{H}_{10}$ which has 2 isomers. Pentane, $\mathrm{C}_{5} \mathrm{H}_{12}$ has 3 and $\mathrm{C}_{6} \mathrm{H}_{14}$ has 5 while $\mathrm{C}_{7} \mathrm{H}_{16}$ has 9 isomers. As the number of carbon atoms increases more and more, the number of isomers increases almost expnentially ${ }^{1-4}$. For instance $\mathrm{C}_{10} \mathrm{H}_{22}$ has 75, $\mathrm{C}_{11} \mathrm{H}_{24}(159), \quad \mathrm{C}_{13} \mathrm{H}_{28}(802), \quad \mathrm{C}_{15} \mathrm{H}_{32}(4,347)$, $\mathrm{C}_{17} \mathrm{H}_{36}(24,894)$ and $\mathrm{C}_{20} \mathrm{H}_{42}(366,319)$. There is no simple formula of generating the number of isomers as the number of carbon atoms increases.

## The number of isomers of hydrocarbons

The number of isomers have been determined by using sophisticated mathematical generating functions ${ }^{1-5}$.

An example of a generating function in an algebraic form is ${ }^{3}$ :

$$
\begin{aligned}
& C(z)=1 z+1 z^{3}+1 z^{4}+2 z^{5}+2 z^{6}+6 z^{7}+9 z^{8}+20 z^{9}+37 z^{10}+ \\
& 86 z^{11}+181 z^{12}+422 z^{13}+\ldots \\
& B(z)=0 z+1 z^{2}+0 z^{3}+1 z^{4}+1 z^{5+3} 27+9 z^{8}+15 z^{9}+38 z^{10} \\
& +73 z^{11}+174 z^{12}+380 z^{13}+\ldots \\
& F(z)=C(z)+B(z)=1 z+1 z^{2}+1 z^{3}+2 z^{4}+3 z^{5}+5 z^{6} \\
& +9 z^{7}+18 z^{8}+35 z^{9}+75 z^{10}+159 z^{11}+802 z^{13}+. .
\end{aligned}
$$

The number of isomers of the alkane series correspond to the coefficients of the formula series.

This work has been extended to include the number of isomers for alkene and alkyne series ${ }^{1-}$ ${ }^{2}$. These have been given in Table 2 for the series $\mathrm{C}_{1}$ to $\mathrm{C}_{20}$.

Ideally, the number of alkane isomers of given molecular formula can be deduced from sketching all the possible isomers guided by the valence principle. This can readily be done for $\mathrm{C}_{4} \mathrm{H}_{10}$ to $\mathrm{C}_{6} \mathrm{H}_{14}$. However from $\mathrm{C}_{7} \mathrm{H}_{16}$ upwards, the exercise becomes more and more difficult.
$\mathrm{C}_{4} \mathrm{H}_{20}$ isomers
1
 ??
1

2

3
 ?
$\mathrm{C}_{6} \mathrm{H}_{34}$




2


Fig. 1: Isomers of $\mathrm{C}_{4} \mathrm{H}_{10}, \mathrm{C}_{5} \mathrm{H}_{12}$, and $\mathrm{C}_{6} \mathrm{H}_{14}$
Table 1: The number of isomers ( NI$)^{1-2}$ and K-values of selected series of hydrocarbons

| $\mathrm{C}_{\mathrm{n}} \mathrm{H}_{\mathrm{m}}$ | MF m =2 | NI | K | $\begin{aligned} & \text { MF } \\ & m=4 \end{aligned}$ | NI | K | MF $m=6$ | NI | K |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{n}=1$ |  |  |  | $\mathrm{CH}_{4}$ | 1 | 0 |  |  |  |
| 2 | $\mathrm{C}_{2} \mathrm{H}_{2}$ | 1 | 3 | $\mathrm{C}_{2} \mathrm{H}_{4}$ | 1 | 2 | $\mathrm{C}_{2} \mathrm{H}_{6}$ | 1 | 1 |
| 3 | $\mathrm{C}_{3} \mathrm{H}_{2}$ | 0 | 5 | $\mathrm{C}_{3} \mathrm{H}_{4}$ | 2 | 4 | $\mathrm{C}_{3} \mathrm{H}_{6}$ | 2 | 3 |
| 4 | $\mathrm{C}_{4} \mathrm{H}_{2}$ | 1 | 7 | $\mathrm{C}_{4} \mathrm{H}_{4}$ | 2 | 6 | $\mathrm{C}_{4} \mathrm{H}_{6}$ | 4 | 5 |
| 5 | $\mathrm{C}_{5} \mathrm{H}_{2}$ | 0 | 9 | $\mathrm{C}_{5} \mathrm{H}_{4}$ | 4 | 8 | $\mathrm{C}_{5} \mathrm{H}_{6}$ | 6 | 7 |
| 6 | $\begin{aligned} & \mathrm{C}_{6} \mathrm{H}_{2} \\ & \mathrm{~m}=8 \end{aligned}$ | 1 | 11 | $\begin{aligned} & \mathrm{C}_{6} \mathrm{H}_{4} \\ & \mathrm{~m}=10 \end{aligned}$ | 5 | 10 | $\begin{aligned} & \mathrm{C}_{6} \mathrm{H}_{6} \\ & \mathrm{~m}=12 \end{aligned}$ | 15 | 9 |
| 3 | $\mathrm{C}_{3} \mathrm{H}_{8}$ | 1 | 2 |  |  |  |  |  |  |
| 4 | $\mathrm{C}_{4} \mathrm{H}_{8}^{\circ}$ | 3 | 4 | $\mathrm{C}_{4} \mathrm{H}_{10}$ | 2 | 3 |  |  |  |
| 5 | $\mathrm{C}_{5} \mathrm{H}_{8}$ | 9 | 6 | $\mathrm{C}_{5} \mathrm{H}_{10}$ | 5 | 5 |  | $3$ | 4 |
| 6 | $\begin{aligned} & \mathrm{C}_{6} \mathrm{H}_{8} \\ & \mathrm{~m}=14 \end{aligned}$ | 22 | 8 | $\mathrm{C}_{6} \mathrm{H}_{10}$ | 23 | 7 | $\mathrm{C}_{6} \mathrm{H}_{12}$ | 13 | 6 |
| 6 | $\mathrm{C}_{6} \mathrm{H}_{14}$ | 5 | 5 |  |  |  |  |  |  |

MF = Molecular formula, $\mathrm{NI}=$ Number of Isomers.

## Application of $\mathbf{k}$ formula to hydrocarbons

The use of K formula has been found to be of great assistance in constructing molecular geometries which obey the octet rule and/or the eighteen electron rule ${ }^{6-7}$. The K formula is given by $K=1 / 2 \quad(E-V)$,
where K is the number of bonds linking the atoms that obey the octet rule, $E$ is the sum of octet electrons surrounding the isolated atoms that obey the octet rule and $V$ is the sum of the valence electrons contributed by all the atoms in a molecular formula in question. This is illustrated by the following examples.

## Calculation of the $\mathbf{k}$ values

Each of the 4 carbon atoms obey the octet rule. Hence the sum of octet electrons will be $\mathrm{E}=$ $4 \times 8=32$. The sum of the valence electrons $\mathrm{V}=$ $4 \times 4+1 \times 10=26$. Therefore the $K$ value will be given by $K=1 / 2(32-26)=3$. Thus all the skeletal $C$ atoms of isomers of $\mathrm{C}_{4} \mathrm{H}_{10}$ will be joined by 3 bonds. $A$
similar calculation for $\mathrm{C}_{5} \mathrm{H}_{12}$ gives a K value of 4 while $\mathrm{C}_{6} \mathrm{H}_{14}$ and $\mathrm{C}_{7} \mathrm{H}_{16}$ have K values of 5 and 6 respectively. The geometries of the corresponding isomers are given in Figures 1 and 2. The simple calculation is easily extended to alkenes and alkynes. For instance, $\mathrm{C}_{2} \mathrm{H}_{4}, \mathrm{~K}=1 / 2(16-12)=2$ and $K$ for $\mathrm{C}_{2} \mathrm{H}_{2}$ is given by $\mathrm{K}=1 / 2(16-10)=3$. Thus, $\mathrm{C}_{2} \mathrm{H}_{4}$ has a double bond while $\mathrm{C}_{2} \mathrm{H}_{2}$ has a triple bond. Tables 1 and 2 show the number of isomers ( NI$)^{1-2}$ and the calculated K-values of a limited selected series of hydrocarbons.

## Sketching the isomers

The sketching the isomers of a given molecular formula is easily done by using the corresponding K value. This has been illustrated for the alkanes $\mathrm{C}_{4} \mathrm{H}_{10}$ to $\mathrm{C}_{7} \mathrm{H}_{16}$ given in Figures 1 and 2. This has been extended to cover other hydrocarbons systems with isomers both acyclic and cyclic ones for $\mathrm{C}_{3} \mathrm{H}_{4}, \mathrm{C}_{4} \mathrm{H}_{4}$, and $\mathrm{C}_{5} \mathrm{H}_{6}$ taken as examples. The selected sketched isomers are given in Figures 3-5.

Table 2: The number of isomers(NI) 1-2 and the $K$ values of alkanes, alkenes and alkynes for the hydrocarbons systems $C_{1}-C_{20}$

| n | Alkanes $\mathrm{C}_{\mathrm{n}} \mathrm{H}_{2 n+2}$ | K | NI | Alken $\mathrm{C}_{\mathrm{n}} \mathrm{H}_{2 n}$ | K | NI | Alkynes $\mathrm{C}_{\mathrm{n}} \mathrm{H}_{2 n-2}$ | K | NI |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\mathrm{CH}_{4}$ | 0 | 1 | $\mathrm{CH}_{2}$ |  |  | C |  |  |
| 2 | $\mathrm{C}_{2} \mathrm{H}_{6}$ | 1 | 1 | $\mathrm{C}_{2} \mathrm{H}_{4}$ | 2 | 1 | $\mathrm{C}_{2} \mathrm{H}_{2}$ | 3 | 1 |
| 3 | $\mathrm{C}_{3} \mathrm{H}_{8}$ | 2 | 1 | $\mathrm{C}_{3} \mathrm{H}_{6}$ | 3 | 1 | $\mathrm{C}_{3} \mathrm{H}_{4}$ | 4 | 2 |
| 4 | $\mathrm{C}_{4} \mathrm{H}_{10}$ | 3 | 2 | $\mathrm{C}_{4} \mathrm{H}_{8}$ | 4 | 3 | $\mathrm{C}_{4} \mathrm{H}_{6}$ | 5 | 4 |
| 5 | $\mathrm{C}_{5} \mathrm{H}_{12}$ | 4 | 3 | $\mathrm{C}_{5} \mathrm{H}_{10}$ | 5 | 5 | $\mathrm{C}_{5} \mathrm{H}_{8}$ | 6 | 9 |
| 6 | $\mathrm{C}_{6} \mathrm{H}_{14}$ | 5 | 5 | $\mathrm{C}_{6} \mathrm{H}_{12}$ | 6 | 13 | $\mathrm{C}_{6} \mathrm{H}_{10}$ | 7 | 23 |
| 7 | $\mathrm{C}_{7} \mathrm{H}_{16}$ | 6 | 9 | $\mathrm{C}_{7} \mathrm{H}_{14}$ | 7 | 27 | $\mathrm{C}_{7} \mathrm{H}_{12}$ | 8 | 58 |
| 8 | $\mathrm{C}_{8} \mathrm{H}_{18}$ | 7 | 18 | $\mathrm{C}_{8} \mathrm{H}_{16}$ | 8 | 66 | $\mathrm{C}_{8} \mathrm{H}_{14}$ | 9 | 152 |
| 9 | $\mathrm{C}_{9} \mathrm{H}_{20}$ | 8 | 35 | $\mathrm{C}_{9} \mathrm{H}_{18}$ | 9 | 153 | $\mathrm{C}_{9} \mathrm{H}_{16}$ | 10 | 400 |
| 10 | $\mathrm{C}_{10} \mathrm{H}_{22}$ | 9 | 75 | $\mathrm{C}_{10} \mathrm{H}_{20}$ | 10 | 377 | $\mathrm{C}_{10} \mathrm{H}_{18}$ | 11 | 1072 |
| 11 | $\mathrm{C}_{11} \mathrm{H}_{24}$ | 10 | 159 | $\mathrm{C}_{11} \mathrm{H}_{22}$ | 11 | 914 | $\mathrm{C}_{11} \mathrm{H}_{20}$ | 12 | 2876 |
| 12 | $\mathrm{C}_{12} \mathrm{H}_{26}$ | 11 | 355 | $\mathrm{C}_{12} \mathrm{H}_{24}$ | 12 | 2281 | $\mathrm{C}_{12} \mathrm{H}_{22}$ | 13 | 7783 |
| 13 | $\mathrm{C}_{13} \mathrm{H}_{28}$ | 12 | 802 | $\mathrm{C}_{13} \mathrm{H}_{26}$ | 13 | 5690 | $\mathrm{C}_{13} \mathrm{H}_{24}$ | 14 | 21099 |
| 14 | $\mathrm{C}_{14} \mathrm{H}_{30}$ | 13 | 1858 | $\mathrm{C}_{14} \mathrm{H}_{28}$ | 14 | 14397 | $\mathrm{C}_{14} \mathrm{H}_{26}$ | 15 | 57447 |
| 15 | $\mathrm{C}_{15} \mathrm{H}_{32}$ | 14 | 4347 | $\mathrm{C}_{15} \mathrm{H}_{30}$ | 15 | 36564 | $\mathrm{C}_{15} \mathrm{H}_{28}$ | 16 | 156686 |
| 16 | $\mathrm{C}_{16} \mathrm{H}_{34}$ | 15 | 10359 | $\mathrm{C}_{16} \mathrm{H}_{32}$ | 16 | 93650 | $\mathrm{C}_{16} \mathrm{H}_{30}$ | 17 | 428438 |
| 17 | $\mathrm{C}_{17} \mathrm{H}_{36}$ | 16 | 24894 | $\mathrm{C}_{17} \mathrm{H}_{34}$ | 17 | 240916 | $\mathrm{C}_{17} \mathrm{H}_{32}$ | 18 | 1173253 |
| 18 | $\mathrm{C}_{18} \mathrm{H}_{38}$ | 17 | 60523 | $\mathrm{C}_{18} \mathrm{H}_{36}$ | 18 | 623338 | $\mathrm{C}_{18} \mathrm{H}_{34}$ | 19 | 3218346 |
| 19 | $\mathrm{C}_{19} \mathrm{H}_{40}$ | 18 | 148284 | $\mathrm{C}_{19} \mathrm{H}_{38}$ | 19 | 1619346 | $\mathrm{C}_{19} \mathrm{H}_{36}$ | 20 | 8839226 |
| 20 | $\mathrm{C}_{20} \mathrm{H}_{42}$ | 19 | 366319 | $\mathrm{C}_{20} \mathrm{H}_{40}$ | 20 | 4224993 | $\mathrm{C}_{20} \mathrm{H}_{38}$ | 21 | 24307593 |



Fig. 2: Isomers of $\mathrm{C}_{7} \mathrm{H}_{16}$

## Isomers of $\mathrm{C}_{3} \mathrm{H}_{4}$

The possible isomers of $\mathrm{C}_{3} \mathrm{H}_{4}(\mathrm{~K}=4, \mathrm{NI}=$ 2) formula are shown in Figure 3.
isomers of $\mathrm{C}_{3} \mathrm{H}_{4}$ are 2 . However, there is one more isomer which is cyclic. It should be emphasized that Tables 1 and 2 gives only the number of acyclic isomers ${ }^{1-2}$.

As can be seen, the number of acyclic
i. $\mathrm{C} \equiv \mathrm{C}-\mathrm{C} \rightarrow \mathrm{HC} \equiv \mathrm{C}-\mathrm{CH}_{3} \quad \rightarrow \bullet$ -
ii. $\mathrm{C}=\mathrm{C}=\mathrm{C} \rightarrow \mathrm{H}_{2} \mathrm{C}=\mathrm{C}=\mathrm{CH}_{2} \quad \rightarrow \quad=$
iii.


Fig. 3: Isomers of $\mathrm{C}_{3} \mathrm{H}_{4}$

## Isomers of $\mathrm{C}_{4} \mathrm{H}_{4}$

The second example is $\mathrm{C}_{4} \mathrm{H}_{4}(\mathrm{~K}=6$ ， NI $=2$ ）．The selected possible isomers are shown in Figure 4.

In this example，two ways of sketching the isomers have been shown in such a way as to emphasize the significance of using the K value．In
addition to the 2 isomers as predicted from the Formula Periodic Table ${ }^{1}$ ，there are more cyclic ones that can be drawn as indicated in Figure 4.

Isomers of $\mathrm{C}_{5} \mathrm{H}_{6}, \mathrm{~K}=7$
The $\mathrm{C}_{5} \mathrm{H}_{6}$ has 6 acyclic isomers ${ }^{1}$ and a K value of 7 ．The selected isomers of $\mathrm{C}_{5} \mathrm{H}_{6}$ are shown in Figure 5.
i．

ii． $\mathrm{H}_{2} \mathrm{C}=\mathrm{C}=\mathrm{C}=\mathrm{CH}_{2}$
iii．

iv．

v


Fig．4：Selected isomers of $\mathrm{C}_{4} \mathrm{H}_{4}$
i． $\mathrm{C} \equiv \mathrm{C}-\mathrm{C}=\mathrm{C}-\mathrm{C} \rightarrow \mathrm{HC} \equiv \mathrm{C}-\mathrm{CH}=\mathrm{CH}-\mathrm{CH}_{3} \rightarrow * \pm$－$\rightarrow=\bullet$－
ii． $\mathrm{C}-\mathrm{C}=\mathrm{C}=\mathrm{C}=\mathrm{C} \rightarrow *$＊＊＊＊＊＊
iii． $\mathrm{C}=\mathrm{C}-\mathrm{C}=\mathrm{C}=\mathrm{C} \rightarrow * *$＊＊$=$＊ ＊
h． $\mathrm{CEC}-\mathrm{C}-\mathrm{C}=\mathrm{C} \rightarrow * \equiv *$－$\rightarrow *$
v． $\mathrm{C}=\mathrm{C}-\mathrm{C} \equiv \mathrm{C}-\mathrm{C} \rightarrow *=*$ ー＊ミー＊
vi．$\underset{\mathrm{c}}{\mathrm{C} \equiv \mathrm{C}-\mathrm{C}-\mathrm{c}} \rightarrow \quad * \pm *$－
vii．

viii．


Fig．5：Selected isomers of $\mathrm{C}_{5} \mathrm{H}_{6}$

## CONCLUSION

The use of $K$ value greatly simplifies the construction of isomers of a given molecular formula of a hydrocarbon. The method is readily extended to other organic compounds with other common elements such as $\mathrm{O}, \mathrm{N}$ and S . It is hoped that by popularizing the use of $K$ value, the teaching and understanding of structural isomerism and
molecular geometry particularly at undergraduate level will immensely be enhanced.

## ACKNOWLEDGEMENTS

I would like to thank Prof. D. J. Klein of the University of Texas A\&M at Galveston and Prof. N. J. A. Sloane of the Information Sciences Research, AT\& T Shannon Lab, USA for the invaluable communications regarding the work.

## REFERENCES

1. Bytautas, L. and Klein, D. J. Croatica Chemica Acta, 73(2): 331(2000).
2. Bytautas, L. , Klein, D. J., Schmalz, T. G. New J. Chem. 24: 329 (2000).
3. Rains, E. M. and Sloane, N. J.A. Journal of Integer sequences, 2: Article 99.1.1(1999).
4. J-L Faulon, D. P. Visco, Jr, and Diana Roe, Enumerating Molecules, Sandia Report, 85
(2004).
5. Private communications with Profs. N. J. A. Sloane and D. J. Klein
6. Kiremire, E. M. R. Oriental Journal of Chemistry, 22(1): 29(2006 )
7. Kiremire, E M R and Kiremire, E B B Materials Science Research India, 4(1): 09 (2007)
